Marking Scheme Form 5 Paper 2 (2013)

SECTION A

1. Solve the following simultaneous equations:

$x^2 + y^2 = xy + 7$

$2x - y = 5$ [5 marks]

$x^2 + y^2 = xy + 7$

$2x - y = 5$

$y = 2x + 5$

$x^2 + (2x+5)^2 = x(2x+5) + 7$ ← (1)

$x^2 + 4x^2 + 20x + 25 = 2x^2 + 5x + 7$

$3x^2 + 15x + 18 = 0$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$ ← (1)

$x = -3$ or $x = -2$ ← (1)

For $x = -3$

$y = 2(-3) + 5 = -1$

For $x = -2$ ← (1) Working to show finding the values of y or x

$y = 2(-2) + 5 = 1$

$x = -2, y = 1$ or $x = -3, y = -1$ ← (1)

2. a) The quadratic equation $x^2 - 6x + 7 = m(2x-3)$ has two equal roots. Find the possible values of m. [3 marks]

b) Hence, determine the stationary point and determine the axis of symmetry for the above equation. [3 marks]

$x^2 - 6x + 7 = m(2x-3)$

$x^2 - 6x + 7 - 2mx + 3m = 0$

$x^2 + (-6-2m)x + 7 + 3m = 0$ ← (1)

$b^2 - 4ac = 0$

$(-6-2m)^2 - 4(1)(7+3m) = 0$

$36 + 24m + 4m^2 - 28 - 12m = 0$

$4m^2 + 12m + 8 = 0$ ← (1)

$m^2 + 3m + 2 = 0$

$(m+1)(m+2) = 0$

$m = -1$ or $m = -2$ ← (1)

(1) One mark is given only if both equations of axis of symmetry are shown

For $m = -1$

$x^2 + (-6-2(-1)x + 7 + 3(-1)) = 0$

$x^2 + (-6+2)x + 7 - 3) = 0$

$x^2 - 4x + 4 = 0$

$\frac{dy}{dx} = 2x - 4 = 0$

Maximum point (2,0) ← (1)

Axis of symmetry $x = 2$

For $m = -2$

$x^2 + (-6-2(-2)x + 7 + 3(-2)) = 0$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

Maximum point (1,0) ← (1)

Axis of symmetry: $x = 1$

3. A closed rectangular box is made of very thin sheet metal, and its length is three times its width. If the volume of the box is 288 cm^3, show that its surface area is equal to $\frac{768}{x}+6x^2$ cm^2, where x cm is the width of the box. [3 marks]

Find by differentiation the dimension of the box of least surface area. [3 marks]

Let y be the height of the cuboid.

Volume $= y\times(x)\times(3x)$

$3x^2y = 288$

$y=\frac{288}{3x^2}$ or $\frac{96}{x^2}$ ← (1)

Surface area, A $=2(3x^2)+2(3x)(\frac{288}{3x^2})+2(x)(\frac{288}{3x^2})$

$A=6x^2+\frac{576}{x}+\frac{576}{3x}$ or $\left(\frac{192}{x}\right)$ ← (1)

$A=6x^2+\frac{768}{x}$

$\frac{dA}{dx}=12x-768x^{-2}$ ← (1)

For maximum or minimum value, $\frac{dA}{dx}=0$

$12x-\frac{768}{x^2}=0$ ← (1)

$12x^3=768$

$x^3=64$

$x=4$ ← (1)

Dimension of cuboid $4\times12\times6$ ← (1)

4. A set of data which consists of 15 numbers has a mean of 12 and a standard deviation of 3.

a) For the set of data, find
   i) the sum of the numbers,
   ii) the sum of squares of the numbers [3 marks]

b) Another set of data which consists of 5 numbers with a mean of 11 and a variance of 8 is added to the original set of data. For the combined set of data, find
   i) the new mean
   ii) the new standard deviation [5 marks]

$n=15,\ \bar{x}=12,\ sd=3$

$\frac{\sum x}{15}=12$

$\sum x=180$ ← (1)

$3^2=\frac{\sum x^2}{15}-12^2$ ← (1)

$135=\sum x^2-2160$

$\sum x^2=2295$ ← (1)

$n=5,\ \bar{x}=11,$ variance $=8$

$\frac{\sum x}{5}=11$

$\sum x=55$ ← (1)

variance $=\frac{\sum x^2}{5}-121=8$ (1) →

$\sum x^2-605=40$

$\sum x^2=645$ ← (1)

$n=20,$

$\frac{180+55}{20}=\bar{x}_{new}$

$\bar{x}_{new}=11.75$ ← (1)

variance $=\frac{645+2295}{20}-11.75^2$

variance $=147-138.0625$

variance $=8.9375$

standard deviation $=\sqrt{8.9375}$

$sd=2.9896$ ← (1)

5. ***Diagram 1*** shows a triangle *OXY*. The straight line *AY* intersects the straight line *XB* at *C*. It is given that $\overrightarrow{OX} = \underset{\sim}{x}$, $\overrightarrow{OY} = \underset{\sim}{y}$, $OA = \frac{1}{3}OX$ and $OB = BY$

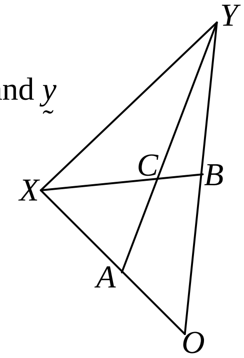


Diagram 1

a) Express each of the following vectors in terms of $\underset{\sim}{x}$ and $\underset{\sim}{y}$

i) $\overrightarrow{AB}$ ii) $\overrightarrow{BX}$ iii) $\overrightarrow{AY}$ [5 marks]

b) Given that $\overrightarrow{BC} = h\overrightarrow{BX}$ and $\overrightarrow{AC} = k\overrightarrow{AY}$, find the value of *h* and of *k*. [4 marks]

$\overrightarrow{OX} = \underset{\sim}{x}$, $\overrightarrow{OY} = \underset{\sim}{y}$, $OA = \frac{1}{3}OX$ and $OB = BY$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ ← 1

$\overrightarrow{AB} = -\frac{1}{3}\underset{\sim}{x} + \frac{1}{2}\underset{\sim}{y}$ ← 1

$\overrightarrow{BX} = \overrightarrow{BO} + \overrightarrow{OX}$ ← 1

$\overrightarrow{BX} = -\frac{1}{2}\underset{\sim}{y} + x$

$\overrightarrow{AY} = \overrightarrow{AO} + \overrightarrow{OY}$ ← 1

$\overrightarrow{AY} = -\frac{1}{3}\underset{\sim}{x} + y$ ← 1

$\overrightarrow{BC} = h\overrightarrow{BX}$ and $\overrightarrow{AC} = k\overrightarrow{AY}$

$\overrightarrow{BO} + \overrightarrow{OC} = h\left(\overrightarrow{BO} + \overrightarrow{OX}\right)$

$-\overrightarrow{OB} + h\overrightarrow{OB} + \overrightarrow{OC} = h\overrightarrow{OX}$

$\overrightarrow{OC} = \overrightarrow{OB} - h\overrightarrow{OB} + h\overrightarrow{OX}$

$\overrightarrow{OC} = \frac{1}{2}\underset{\sim}{y} - \frac{h}{2}\underset{\sim}{y} + h\underset{\sim}{x}$

$\overrightarrow{OC} = \left(\frac{1-h}{2}\right)\underset{\sim}{y} + h\underset{\sim}{x}$ .... (1) ← 1

$\overrightarrow{AC} = k\overrightarrow{AY}$

$\overrightarrow{AO} + \overrightarrow{OC} = k\left(\overrightarrow{AO} + \overrightarrow{OY}\right)$

$\overrightarrow{OC} = \overrightarrow{OA} - k\overrightarrow{OA} + k\overrightarrow{OY}$

$\overrightarrow{OC} = (1-k)\frac{1}{3}\underset{\sim}{x} + k\underset{\sim}{y}$ .... (2) ← 1

By comparing

$h = (1-k)\frac{1}{3}$ $k = \left(\frac{1-h}{2}\right)$

$3h + k = 1$.............(3)

$2k + h = 1$..............(4)

From, (3) $k = 1 - 3h$ sub. into (4)

$2(1-3h) + h = 1$

$2 - 6h + h = 1$

$2 - 5h = 1$

$5h = 1$

$h = \frac{1}{5}$ ← 1

For $h = \frac{1}{5}$

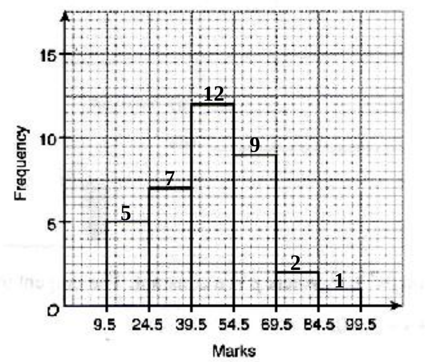
$k = 1 - 3h$

$k = 1 - 3\left(\frac{1}{5}\right)$

$k = 1 - \frac{3}{5}$

$k = \frac{2}{5}$ ← 1

6. The histogram below shows the marks obtained by a Form 5 class of students in an Additional Mathematics test.



a) Without drawing an ogive, calculate the median mark [3 marks]

b) Calculate the standard deviation of the marks [3 marks]

Class intervals	Frequency	Cumulative Frequency	x	fx	fx^2
10 – 24	5	5	17	85	1445
25 – 39	7	12	32	224	7168
40 – 54	12	24	47	564	26508
55 – 69	9	33	62	558	34596
70 – 84	2	35	77	154	11858
85 - 99	1	36	92	92	8464
	$\sum f = 36$			$\sum fx = 1677$	$\sum fx^2 = 90039$

(1)

Median = $L + \left(\dfrac{\frac{N}{2} - F}{f} \right) c$

Median = $39.5 + \left(\dfrac{18 - 12}{12} \right) 15$ (1)

$39.5 + 7.5$

47 (1)

$\bar{x} = \dfrac{\sum fx}{n}$

$\bar{x} = \dfrac{1602}{36}$

$\bar{x} = 46.6$ (actually is 46.58) (1)

$Sd = \sqrt{\dfrac{\sum fx^2}{n} - \bar{x}^2}$

$Sd = \sqrt{\dfrac{90039}{36} - (46.6)^2}$ (1)

$sd = \sqrt{2501.08 - 2171.56}$

$sd = \sqrt{329.52}$

$sd = 18.1527$ (1)

SECTION B

7. Use the graph paper to answer this question

Table 1 shows the values of two variables, x and y, obtained from an experiment

The variables x and y are related by the equation $y = \dfrac{px + 1}{qx^2}$, where p and q are constants.

x	1	2	3	4	5	6
y	2.601	0.551	0.194	0.089	0.040	0.017

Table 1

a) Based on ***table 1***, construct a suitable table for the values of x^2y [1 mark]

b) Plot x^2y against x, using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 0.5 unit on the x^2y - axis
Hence, ***draw the line of best fit***. [3 marks]

c) Use the graph drawn to give the best-estimated value of
i) y when $x = 2.5$
ii) p
iii) q [6 marks]

$$y = \frac{px+1}{qx^2}$$

$$qyx^2 = px+1$$

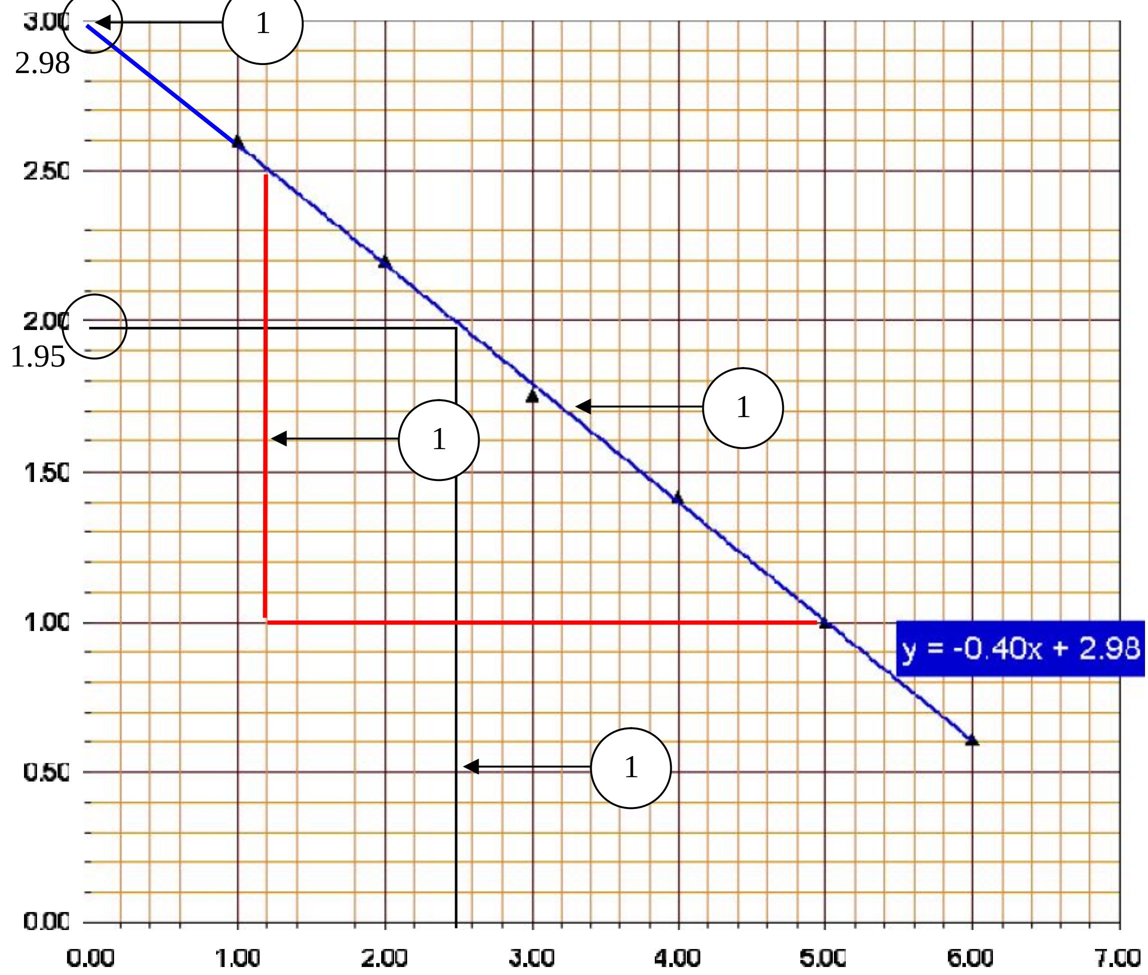
$$yx^2 = \frac{p}{q}x + \frac{1}{q}$$ ← 1

It is best if the values in the table are in four decimals places

x	1	2	3	4	5	6
yx^2	2.601	2.204	1.746	1.424	1	0.612

← 1 *Table 1*

The values of p and q are derived by reading the graph



From the graph,

When x = 2.5 the value of y is $\frac{1.95}{2.5^2} = 0.312 \pm 0.05$ ← 1

The readings on the x-axis must be two decimal places

$\frac{p}{q} = -0.4$ and

$\frac{1}{q} = 2.98$ ← 1

$q = 0.3356$ ← 1

$p = -0.4 \times 0.3356$

$p = -0.1342$ ← 1

8. ***Diagram 2*** shows an equation $y = x(x-4)$, the x-axis, the straight line $y = 5$ and the straight line AB

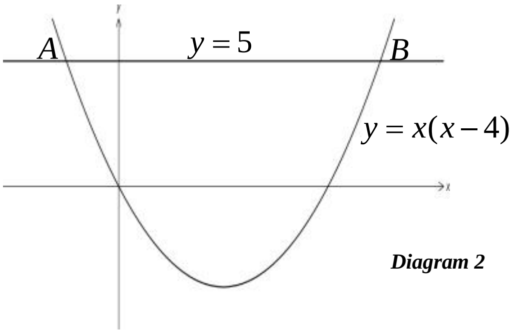


Diagram 2

Find

a) The turning point of the curve $y = x(x-4)$ [2 marks]

b) Determine the axis of symmetry of the curve. [1 mark]

c) The equations of normal at point A and point B [3 marks]

d) Hence or otherwise, determine the point of intersection, D of the normal at point A and normal at the point B [3 marks]

e) What can you say about the position of point D, the midpoint of AB and the turning point of the curve $y = x(x-4)$? [1 mark]

$y = x(x-4)$

$y = x^2 - 4x$

$\frac{dy}{dx} = 2x - 4$ ← 1

For turning point, $\frac{dy}{dx} = 0$

$2x - 4 = 0$

$x = 2$

$y = 2^2 - 4(2)$

$y = 4 - 8$

$y = -4$

Turning point $(2, -4)$ ← 1

Or anyother methods

Thus, the axis of symmetry is $x = 2$ ← 1

$y = x^2 - 4x$

$y = 5$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$x = -1$ or $x = 5$ ← 1

$\frac{dy}{dx} = 2x - 4$

At $x = -1$, at A

$m_A = 2(-1) - 4 = -6$

At $x = 5$, at B

$m_B = 2(5) - 4 = 6$

Equation of normal at A

$y - 5 = \frac{1}{6}(x+1)$

$6y - x = 31$ ← 1

Equation of normal at B

$y - 5 = -\frac{1}{6}(x-5)$ ← 1

$6y + x = 35$

$6y - x = 31 ....(eq1)$

$6y + x = 35 ....(eq2)$

At point D, $(eq2 + eq1)$

$12y = 66$

$y = \frac{11}{2}$ ← 1

From $(eq1)$

$-13 + 6x + 1 = 0$

$6x - 12 = 0$ ← 1

$x = 2$

$D(2, \frac{11}{2})$ ← 1

The three points are collinear
or three points lies
on the axis of symmetry

9. ***Diagram 3*** shows two arcs, PQ and RS, of two concentric circles, with the same centre O. RQ is perpendicular to OS.

Given that $OP = OQ = 5$ cm and $\angle POQ = 1$ radian,

find

a) the perimeter of the shaded region A, [7 marks]

b) the area of the shaded region B. [3 marks]

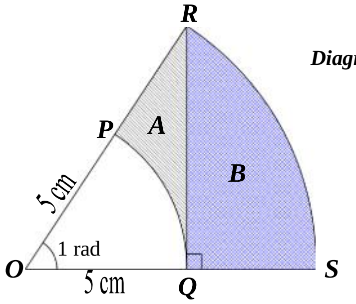


Diagram 3

$1rad = \frac{1}{3.142} \times 180 = 57.2884^{o}$

$\tan(57.2884^{o}) = \frac{RQ}{5}$ ← (1)

$RQ = 5(\tan 57.2884)$

$RQ = 7.7848cm$ ← (1)

$Cos(57.2884^{o}) = \frac{5}{OR}$

$OR = \frac{5}{\cos(57.2884)}$ ← (1)

$OR = 9.2522cm$ ← (1)

$Thus, PR = OR - OP$

$PR = 9.2522 - 5$

$PR = 4.2522cm$ ← (1)

Perimeter of $A = S_{PQ} + QR + RP$ ← (1)

$Perimeter = 5(1) + 7.7848 + 4.2522$

$Perimeter = 17.037cm$ ← (1)

Area of shaded region, B

Area = Area of sector, ORS − Area of ΔORQ

$Area = \frac{1}{2}(9.2522)^{2}(1) - \frac{1}{2}(5)(7.7848)$ ← (1)

$Area = 42.802 - 19.462$ ← (1)

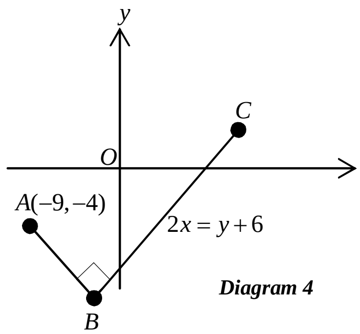
$Area, B = 23.34cm^{2}$ ← (1)

10. *Solutions by scale drawing will not be accepted for this question.*
Diagram 4 shows that the straight lines *AB* and *BC* are perpendicular to each other. The equation of the straight line *BC* is $2x = y + 6$

a) Find
   i) the equation of the straight line *AB* giving your answer in the general form,
   ii) the coordinates of point *B*. [4 marks]

b) The straight line *AB* is extended to point *D* such that *AB* : *BD* = 2 : 3. Calculate the area of triangle *ADO*. [3 marks]

c) A point *M* moves such that the angle *AMB* is always a right-angle. Find the equation of the locus of *M*. [3 marks]



Eq of *BC*; $y = 2x - 6$

$m_{BC} = 2$

Since BC and AB is perpendicular to one another,

$m_{BC} \times m_{AB} = -1$

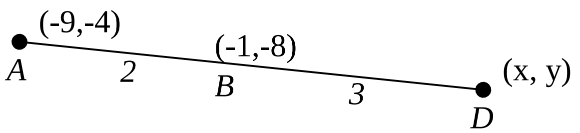
$m_{AB} = -\frac{1}{2}$ ← (1)

Equation of AB through $(-9, -4)$

$y + 4 = -\frac{1}{2}(x + 9)$

$2y + 8 = -x - 9$

$x + 2y + 17 = 0$ ...... (In general form) ← (1)



Coordinate *B*

$x + 2(2x - 6) = -17$

$x + 4x - 12 = -17$

$5x = -5$ ← (1)

$x = -1$

$y = 2(-1) - 6$

$y = -8$

$B(-1, -8)$ ← (1)

$(-1, -8) = \left(\frac{3(-9) + 2(x)}{5}, \frac{3(-4) + 2(y)}{5} \right)$ ← (1)

$-5 = -27 + 2x$ $\quad$ $-40 = -12 + 2y$

$2x = 22$ $\quad$ $2y = -28$

$x = 11$ $\quad$ $y = -14$

$D(11, -14)$ ← (1)

Thus area of triangle *ADO*

$$Area = \frac{1}{2}\begin{bmatrix} -9 & 11 & 0 & -9 \\ -4 & -14 & 0 & -4 \end{bmatrix}$$

$$Area = \frac{1}{2}[126 + 44]$$

$$Area = 85 \text{ unit}^2$$ ← 1

$Let\ M = (x, y),\ A(-9, -4),\ B(-1, -8)$

since $AM \pm MB$, therefore, $m_{AM} \times m_{MB} = -1$ ← 1

$$m_{AM} = \left(\frac{y+4}{x+9}\right)$$

$$m_{MB} = \left(\frac{y+8}{x+1}\right)$$

$$\left(\frac{y+4}{x+9}\right) \times \left(\frac{y+8}{x+1}\right) = -1$$ ← 1

$$y^2 + 8y + 4y + 32 = -x^2 - x - 9x - 9$$

$$x^2 + y^2 + 10x + 12y + 41 = 0$$ ← 1

11. a) ***Diagram 5*** shows a shaded region bounded by the curve $y^2 = x - k$, the y-axis and the straight lines $y = -3$ and $y = 3$. If the area of the shaded region is 30 units^2, find the value of *k*. [4 marks]

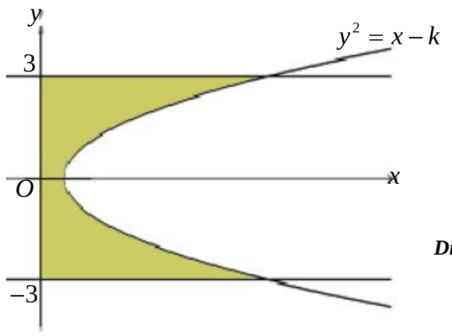


Diagram 5

b) Find the ratio of the volumes if the shaded region is rotated about the y-axis for 180° to that of the shaded region if it is rotated about the x-axis for 180°. [6 marks]

Area bounded by the y-axis

$$\text{Area} = \int_0^3 x\,dy$$

$$x = y^2 + k$$ ← 1

$$\text{Area} = \int_0^3 \left(y^2 + k\right) dy$$

$$\text{Area} = \left[\frac{y^3}{3} + ky\right]_0^3$$ ← 1

$$\text{Area} = 9 + 3k$$

$$9 + 3k = 15$$ ← 1

$$3k = 6$$

$$k = 2$$ ← 1

Volume generated along the y-axis

$$\text{Volume} = ® \int_{-3}^{3} x^2 dy$$

$$x = y^2 + 2$$

$$\text{Volume} = ® \int_{-3}^{3} \left(y^2 + 2\right)^2 dy$$

$$\text{Volume} = ® \int_{-3}^{3} \left(y^4 + 4y^2 + 4\right) dy$$ ← 1

$$\text{Volume} = ® \left[\frac{y^5}{5} + \frac{4y^3}{3} + 4y\right]_{-3}^{3}$$

$$\text{Volume} = ® \left(\frac{243}{5} + 36 + 12 - \left(\frac{-243}{5} - 36 - 12\right)\right)$$ ← 1

Since it is rotated only about 180°,

the volume is halved

$$\text{Volume} = \frac{1}{2}\left(\frac{486}{5} + 72 + 24\right) ® \text{unit}^3$$

$$\text{Volume} = \frac{966®}{10} \text{ unit}^3 \text{ or } 96.6®$$ ← 1

Volume generated along the *x*-axis

$\text{Volume} = ® \int_0^3 y^2 dx$

$y^2 = x - 2$

If $y = 3$; then, $x = 9 + 2 = 11$

If $y = 0$; then, $x = 2$

$\text{Volume} = ® (3)^2 11 - ® \int_2^{11} (x-2)\, dx$ ← (1)

$\text{Volume} = 99® - ® \left[\frac{x^2}{2} - 2x \right]_2^{11}$

$\text{Volume} = 99® - ® \left(\frac{121}{2} - 22 - (2-4) \right)$

$\text{Volume} = 99® - ® \left(\frac{77}{2} - 20 \right)$

$\text{Volume} = 99® - \frac{81}{2}®$

$\text{Volume} = \frac{117}{2}® \text{ or } 58.5® \text{ u}$ ← (1)

Thus the ratio

$\frac{966®}{10} : \frac{117®}{2} = \frac{966 \times 2}{10 \times 117}$

The ratio is 322:195 ← (1)

SECTION C

12. The cost to produce a tin of paint depends on the cost of the raw materials, the production cost and the packaging cost. The table shows the price indices and weightages of those costs.

Cost	I_{2002} (based on the year 1999)	I_{2005} (based on the year 1999)	I_{2005} (based on the year 2002)	Weightages, *w*
Raw materials	175	182	104	*x*
Production	*h*	200	125	5
Packaging	145	*k*	120	2

a) Given that the composite index of the cost to produce a tin of paint for the year 2005 based on the year 2002 is 117.7, find the value of *x*. [3 marks]

b) Given that the price of a tin of paint in the year 2002 was RM 30, calculate its corresponding price in the year 2005. [1 mark]

c) Find the values of

i) *h*,

ii) *k*. [6 marks]

$I_{2005} = \frac{104x + 125(5) + 120(2)}{7 + x} = 117.7$ ← (1)

$\frac{104x + 865}{7 + x} = 117.7$ ← (1)

$104x + 865 = 823.9 + 117.7x$

$865 - 823.9 = (117.7 - 104)x$

$41.1 = 13.7x$

$x = 3$ ← (1)

$\frac{P_{2005}}{P_{2002}} \times 100 = 117.7$

$\frac{P_{2005}}{30} = \frac{117.7}{100}$

$P_{2005} = RM\,35.31$ ← (1)

$$\frac{P_{2002}}{P_{1999}} = \frac{h}{100}$$

$$\frac{P_{2005}}{P_{1999}} = \frac{200}{100}$$ (1)

$$\frac{P_{2005}}{P_{2002}} = \frac{125}{100}$$

$$\frac{P_{2002}}{P_{1999}} = \frac{P_{2002}}{P_{2005}} \times \frac{P_{2005}}{P_{1999}}$$

$$\frac{P_{2002}}{P_{1999}} = \frac{100}{125} \times \frac{200}{100} = \frac{h}{100}$$ (1)

$$h = \frac{200 \times 100}{125}$$

$$h = 160$$ (1)

$$\frac{P_{2002}}{P_{1999}} = \frac{145}{100}$$

$$\frac{P_{2005}}{P_{1999}} = \frac{k}{100}$$ (1)

$$\frac{P_{2005}}{P_{2002}} = \frac{120}{100}$$

$$\frac{P_{2005}}{P_{1999}} = \frac{P_{2005}}{P_{2002}} \times \frac{P_{2002}}{P_{1999}}$$

$$\frac{P_{2002}}{P_{1999}} = \frac{120}{100} \times \frac{145}{100} = \frac{k}{100}$$ (1)

$$k = \frac{120 \times 145 \times 100}{100 \times 100}$$

$$k = 174$$ (1)

13. a) If $m = \log_6 3$ and $n = \log_6 5$, express $\log_2 45$ in terms of m and n. [5 marks]

b) Given $\log_a x = 6$, $\log_b x = 8$ and $\log_{abc} x = 3$, find the value of $\log_c x$ [5 marks]

$$\log_6 3 = \frac{\log_2 3}{\log_2 6} = m \text{ .... eq 1}$$

$$\log_6 5 = \frac{\log_2 5}{\log_2 6} = n \text{ .... eq 2}$$ (1)

From eq1, $\log_2 3 + \log_2 2 = \frac{\log_2 3}{m}$

$$m\log_2 3 + m = \log_2 3$$

$$(1-m)\log_2 3 = m$$

$$\log_2 3 = \frac{m}{1-m}$$ (1)

From eq 2, $\log_2 6 = \frac{\log_2 5}{n}$

$$\log_2 3 + \log_2 2 = \frac{\log_2 5}{n}$$

$$\frac{m}{1-m} + 1 = \frac{\log_2 5}{n}$$

$$\log_2 5 = \frac{n}{1-m}$$ (1)

$$\log_2 45 = \log_2 (5 \times 9)$$

$$\log_2 45 = \log_2 5 + 2\log_2 3$$ (1)

$$\log_2 45 = \frac{n}{1-m} + \frac{2m}{1-m}$$

$$\log_2 45 = \frac{2m+n}{1-m}$$ (1)

$\log_a x = 6$, $\log_b x = 8$ and $\log_{abc} x = 3$. Find $\log_c x$

$$\frac{\log_c x}{\log_c a} = 6 \text{ or } \log_c x = 6\log_c a$$ (1)

$$\frac{\log_c x}{\log_c b} = 8 \text{ or } \log_c x = 8\log_c b$$ (1)

$$\frac{\log_c x}{\log_c abc} = 3$$

$$\log_c x = 3\log_c a + 3\log_c b + 3\log_c c$$ (1)

$$\log_c x = \frac{1}{2}\log_c x + \frac{3}{8}\log_c x + 3$$

$$\left(1 - \frac{1}{2} - \frac{3}{8}\right)\log_c x = 3$$ (1)

$$\left(\frac{16-8-6}{16}\right)\log_c x = 3$$

$$\log_c x = \frac{3 \times 16}{2}$$

$$\log_c x = 24$$ (1)

14. ***Diagram 6*** shows, *AEC* and *BED* are straight lines and *BE* = *ED*. It is given that *AB* = 9 cm, *AD* = 14 cm, *CD* = 15 cm, $\angle AED = 115^{\circ}$ and $\angle ACD = 35^{\circ}$.

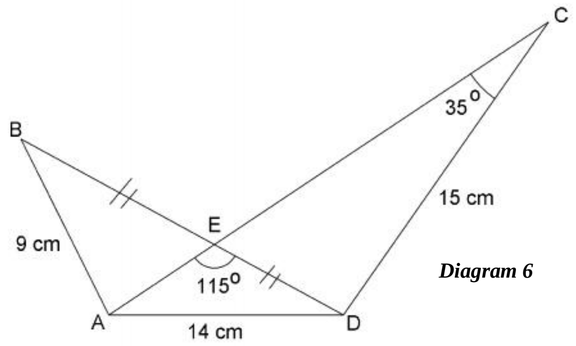


Diagram 6

Calculate

a) the length of *BD*, [4 marks]

b) $\angle BAD$, [2 marks]

c) the area of the whole diagram. [4 marks]

$\frac{\sin 35}{14} = \frac{\sin \overline{\ }}{15}$ ← (1)

$\sin \overline{\ } = 0.6145$

$\overline{\ } = 37.92^{\circ}$ ← (1)

$\frac{\sin 37}{ED} = \frac{\sin 115}{14}$

$ED = \frac{14 \sin 37}{\sin 115}$

$ED = 9.4923$ ← (1)

Therefore, $BD = 2ED$

$BD = 2 \times 9.4923$

$BD = 18.9847cm$ ← (1)

Using cosine rule,

$18.9847^2 = 81 + 196 - 2(9)(14)\cos(\angle BAD)$ ← (1)

$360.0418 = 277 - 252\cos(\angle BAD)$

$\cos(\angle BAD) = -0.32953$

$(\angle BAD) = 109.24^{\circ}$ ← (1)

Area of whole diagram = area of $\Delta BAD + \Delta EDC$

Area $= \frac{1}{2}(9)(14)\sin 109.24 +$ area of ΔEDC

But first find the following:

$\angle CED = 180 - 115 = 65^{\circ}$ ← (1)

Thus, $\angle EDC = 180 - 65 - 35 = 80^{\circ}$

Using sine rule to find the length of *EC*

$\frac{EC}{\sin 80} = \frac{15}{\sin 65}$

$EC = \frac{15 \sin 80}{\sin 65}$

$EC = 16.2992cm$ ← (1)

Therefore, Area $= \frac{1}{2}(9)(14)\sin 109.24 + \frac{1}{2}(16.2992)(15)\sin 35$ ← (1)

Area $= 59.48123 + 70.11627$

Area $= 129.598cm^2$ ← (1)

15. ***Diagram 7*** shows a circular cylinder of height *h* and radius *r* surmounted by a hemisphere of the same radius.

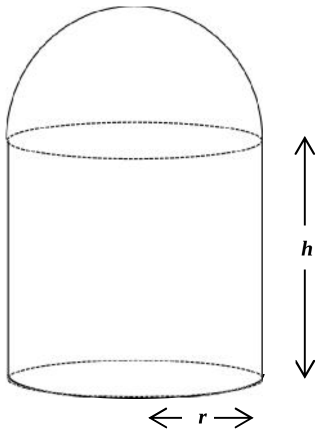


Diagram 7

Express the total surface area *S* of the object and the total volume *V* in terms of *h* and *r*.

[2 marks]

If the total surface area *S* is 20π, express *h* in terms of *r* and hence show that $V = 10®r - \frac{5®r^3}{6}$.

[2 marks]

Find the value of *r* that makes *V* a maximum and calculate the maximum value of *V*, giving your answer in terms of π

[6 marks]

$\text{Surface area } = 2®rh + \frac{1}{2}\left(4®r^2\right) + ®r^2$

$\text{Surface area } = 2®rh + 2®r^2 + ®r^2$

$\text{Surface area } = 2®rh + 3®r^2$ ← 1

$Volume = ®r^2h + \frac{2}{3}®r^3$ ← 1

$2®rh + 3®r^2 = 20®$

$2rh + 3r^2 = 20$

$h = \frac{20-3r^2}{2r}$ ← 1

$Volume = ®r^2\left(\frac{20-3r^2}{2r}\right) + \frac{2}{3}®r^3$

$Volume = \frac{20®r^2 - 3®r^4}{2r} + \frac{2}{3}®r^3$

$Volume = \frac{20®r - 3®r^3}{2} + \frac{2}{3}®r^3$

$Volume = \frac{60®r - 9®r^3 + 4®r^3}{6}$

$Volume = \frac{60®r}{6} - \frac{5®r^3}{6}$

$Volume = 10®r - \frac{5®r^3}{6}$ .... shown ← 1

Workings above must be shown clearly and correctly

$V = 10®r - \frac{5®r^3}{6}$ ← 1

For maximum volume, $\frac{dV}{dr} = 0$

$\frac{dV}{dr} = 10® - \frac{15®r^2}{6}$ ← 1

$10® - \frac{15®r^2}{6} = 0$

$60® - 15®r^2 = 0$ ← 1

$15®r^2 = 60®$

$r^2 = 4$

$r = 2cm$ ← 1

$V = 10®(2) - \frac{5®(2)^3}{6}$

$V = 20® - \frac{40®}{6}$

$V = 20® - \frac{20}{3}®$ ← 1

$V = \frac{40}{3}®$ ← 1

End of Questions