MARKING SCHEME
ADDITIONAL MATHEMATICS PAPER 2
FINAL TRIAL 2008

1. $x + y = 2$

$y = 2 - x$

$sub\ y = 2 - x$ into $x^2 + 5y - 15 = 0$

$x^2 + 5(2-x) - 15 = 0$

$x^2 + 10 - 5x - 15 = 0$

$x^2 - 5x - 5 = 0$

$x = \frac{5 \pm \sqrt{25 - 4(1)(-5)}}{2}$

$x = \frac{5 \pm \sqrt{25+20}}{2} = \frac{5 \pm \sqrt{45}}{2}$

$x = \frac{5+\sqrt{45}}{2}$ or $\frac{5-\sqrt{45}}{2}$

$x = \frac{5+6.708204}{2}$ or $\frac{5-6.708204}{2}$

$x = 5.854$ or -0.8541

2.

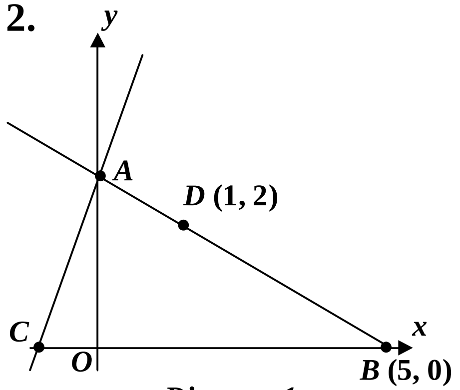


Diagram 1

$m_{BD} = \left(\frac{2-0}{1-5}\right)$

$m_{BD} = -\frac{1}{2}$

$Eq\ of\ BD$

$y - 0 = -\frac{1}{2}(x-5)$

$2y = -x + 5$

$2y + x = 5$

At A, $x = 0;\ y = \frac{5}{2}$

thus, $A(0, \frac{5}{2})$

We know that $AB \perp AC$

$\therefore m_{AC} \times m_{AB} = -1$

$m_{AC}\left(-\frac{1}{2}\right) = -1$

$m_{AC} = 2$

Therefore, equation of AC,

$y - \frac{5}{2} = 2(x - 0)$

$2y - 5 = 2x$

$2y = 2x + 5$

At C, where $y = 0$

$x = -\frac{5}{2}$

$C(-\frac{5}{2}, 0)$

let P be the locus with coordinate (x, y)

Since the distance of PD is always 2 units

thus, $\sqrt{(x-1)^2 + (y-2)^2} = 2$

$(x-1)^2 + (y-2)^2 = 4$

b) $x^2 - 2x + 1 + y^2 - 4y + 4 = 4$

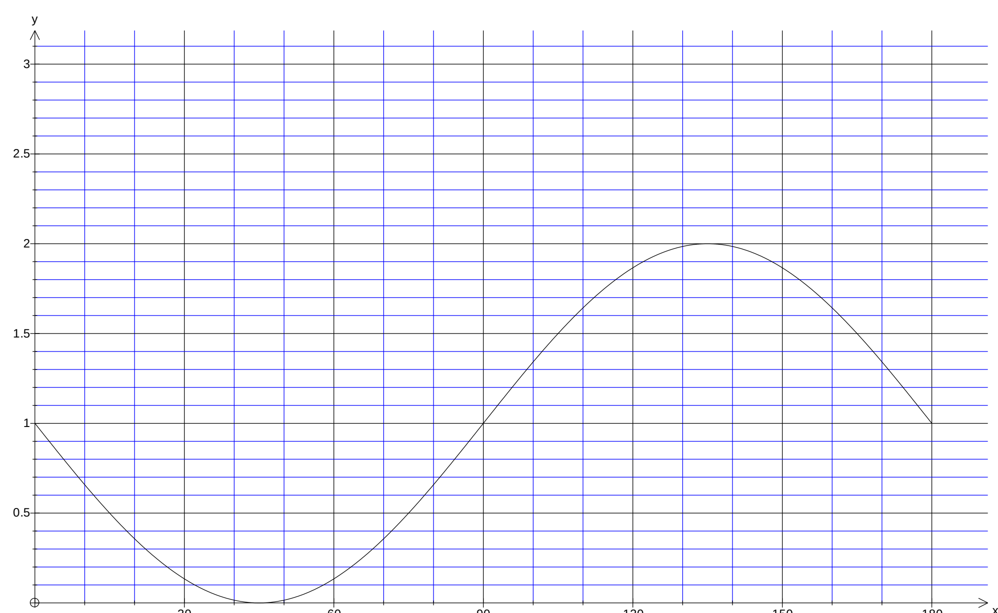
$x^2 + y^2 - 2x - 4y + 1 = 0$

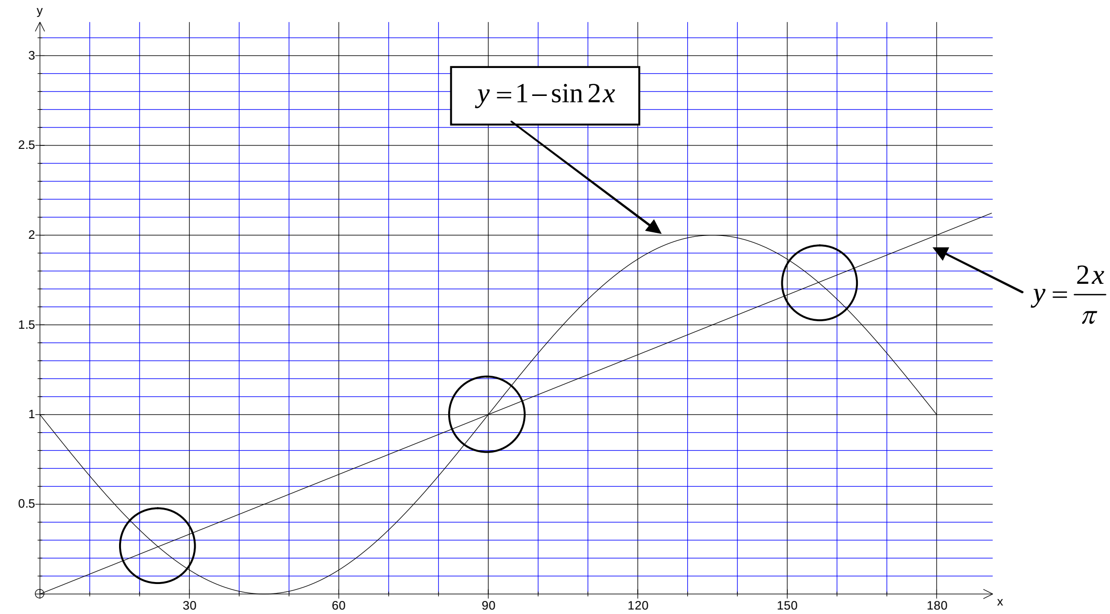
Equation of BD

$2y + x = 5$

$y = -\frac{1}{2}x + \frac{5}{2}$

3.





From $2\pi \sin x \cos x = \pi - 2x$

$$\frac{2\pi \sin x \cos x}{2\pi} = \frac{\pi - 2x}{2\pi}$$

$$\sin x \cos x = \frac{1}{2} - \frac{x}{\pi}$$

$$2\sin x \cos x = 1 - \frac{2x}{\pi}$$

$$\frac{2x}{\pi} = 1 - 2\sin x \cos x$$

$$\frac{2x}{\pi} = 1 - \sin 2x$$

Therefore, the two equations that intersect each other are:

$$y = \frac{2x}{\pi}$$

$$y = 1 - 2\sin x$$

From the graph, there are 3 solutions

5.

Class intervals	*x*	*f*	*fx*	*fx^2*
0-10	5	10	50	250
11-20	15.5	12	186	2883
21-30	25.5	34	867	22108.5
31-40	35.5	12	426	15123
41-50	45.5	12	546	24843
		$\sum f = 80$	$\sum fx = 2075$	$\sum fx^2 = 65207.5$

a) mean, $\bar{x} = \frac{\sum fx}{\sum f}\bar{x}$

$$\bar{x} = \frac{2075}{80}$$

$$\bar{x} = 25.94$$

b) Standard deviation $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$

$$\sigma = \sqrt{\frac{65207.5}{80} - (25.94)^2}$$

$$\sigma = \sqrt{815.094 - 672.884}$$

$$\sigma = \sqrt{142.21} = 11.9252$$

4.

$$\frac{dy}{dx} = 3x^2 + x - 2$$

$$dy = (3x^2 + x - 2)dx$$

$$y = \frac{3x^3}{3} + \frac{x^2}{2} - 2x + c$$

$$y = x^3 + \frac{1}{2}x^2 - 2x + c$$

At $x = 2, y = 5$

$$5 = 2^3 + \frac{1}{2}(4) - 4 + c$$

$$c = 5 - 8 - 2 + 4$$

$$c = -1$$

$$y = x^3 + \frac{1}{2}x^2 - 2x - 1$$

At turning points, $\frac{dy}{dx} = 0$

$$3x^2 + x - 2 = 0$$

$$(3x - 2)(x + 1) = 0$$

$x = \frac{2}{3}$ or $x = -1$

$$\frac{d^2y}{dx^2}=6x+1$$

When $x=\frac{2}{3}$

$\frac{d^2y}{dx^2}>0$ thus, minimum pt.

When $x=-1$

$\frac{d^2y}{dx^2}<0$ thus, maximum pt.

For $x=\frac{2}{3}, y=\frac{8}{27}+\frac{1}{2}\left(\frac{4}{9}\right)-2\left(\frac{2}{3}\right)-1$

$$y=\frac{8}{27}+\frac{2}{9}-\left(\frac{4}{3}\right)-1$$

$$y=\frac{8+6-36-27}{27}$$

$$y=\frac{-49}{27}$$

Minimum pt $(\frac{2}{3},\frac{-49}{27})$ or $(0.6667,-1.8148)$

For $x=-1, y=-1+\frac{1}{2}(1)-2(-1)-1$

$$y=-1+\frac{1}{2}+2-1$$

$$y=\frac{1}{2}$$

maximum pt. $(-1,\frac{1}{2})$

6. $S_n=n(n+1)$

$S_1=T_1=1(1+1)=a$

$T_1=a=2$

$T_2=S_2-S_1$

$T_2=2(2+1)-2$

$T_2=6-2=4$

$\therefore d=4-2=2$

$$\frac{x+2}{x-3}=\frac{3x-4}{x+2}$$

$x^2+4x+4=(3x-4)(x-3)$

$x^2+4x+4=3x^2-9x-4x+12$

$x^2+4x+4=3x^2-13x+12$

$2x^2-17x+8=0$

$(2x-1)(x-8)=0$

$x=\frac{1}{2}$ or $x=8$

Section B

7. $yx^m=k$

$\lg yx^m=\lg k$

$\lg y+m\lg x=\lg k$

$\lg y=-m\lg x+\lg k$

x	*3*	*4*	*5*	*6*	*7*
y	*103*	*87*	*76*	*68*	*62*
lgy	*2.01*	*1.94*	*1.88*	*1.83*	*1.79*
lgx	*0.48*	*0.60*	*0.70*	*0.78*	*0.85*

From the graph, $\lg k=2.29$

$\therefore k=\lg^{-1}(2.29)$

$k=194.98$

From the graph, the gradient,

$$-m=-\frac{0.34}{0.58}$$

$m=0.5862$

When $x=2$, $\lg 2=0.3$

From the graph, the intercept is 0.216

$\lg y=0.216$

$y=1.644$

8.

$AM=MB$

$AO+OM=MO+OB$

$-2a+OM=-OM+6b$

$2OM=2a+6b$

$OM=a+3b$

$AP=AO+OP$

$AP=-2a+\frac{1}{3}(6b)$

$AP=-2a+2b$

$OQ=pOM$

$OQ=p(a+3b)$

$OQ=pa+3pb$

$AQ=kAP$

$AQ=k(-2a+2b)$

$AQ=-2ka+2kb$

$OQ = pa + 3pb$

$AQ = -2ka + 2kb$

$AQ = AO + OQ$

$-2ka + 2kb = -2a + pa + 3pb$

$-2ka + 2kb = (-2 + p)a + 3pb$

By comparing,

$-2k = -2 + p$ and $2k = 3p$

$p + 2k = 2$ equation 1

$k = \frac{3}{2}p$ equation 2

sub 2 into 1

$p + 2(\frac{3}{2}p) = 2$

$4p = 2$

$p = \frac{1}{2}$

$k = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

$k = \frac{3}{4}$

9.

Given that OA = r = 8 cm

$S_{AB} = r\theta$

$S_{AB} = 8 \times 0.9$

$S_{AB} = 7.2$ cm

0.9 radian is $\frac{0.9}{3.142}(180) = 51.56^o$

$\sin 51.56 = \frac{AD}{8}$

$AD = 8(\sin 51.56)$

$AD = 6.2660$ cm

$OD = \sqrt{64 - (6.266)^2}$

$OD = \sqrt{64 - 39.263}$

$OD = 4.9736$

$DB = 8 - 4.9736 = 3.0264$ cm

Therefore, the perimeter

X is 7.2+6.266+3.0264=16.4924 cm

$Cos 51.56 = \frac{8}{OC}$

$OC = \frac{8}{0.6217} = 12.8679cm$

$DC = 12.8679 - 4.936 = 7.9319cm$

$Area\ Y = \frac{1}{2}(12.8679)(8)\sin 51.56 -$

$(\frac{1}{2}(64)(0.9))$

$Y = 40.3156 - 28.8$

$Y = 40.3156 - 12.42$

$Y = 11.5156$ cm^2

10.

$y = \frac{1}{4x-2}$

$y = (4x-2)^{-1}$

$\frac{dy}{dx} = -1(4x-2)^{-2}$

$\frac{dy}{dx} = -\frac{1}{(4x-2)^2}$

At A, $x = 1$

$m_t = -\frac{1}{4}$

$y = \frac{1}{2}$

$A(1, \frac{1}{2})$

Equation of Normal

$y - \frac{1}{2} = 4(x-1)$

$2y - 1 = 4x - 4$

$2y - 4x = -3$

At P, when $y = 0$

$x = \frac{3}{4}$

$P(\frac{3}{4}, 0)$

$Volume = \pi\int_1^k \frac{1}{(4x-2)^2}dx = \frac{\pi}{10}$

$V = \pi\int_1^k (4x-2)^{-2}dx = \frac{\pi}{10}$

$V = \pi\left[\frac{(4x-2)^{-1}}{-4}\right]_1^k = \frac{\pi}{10}$

$V = \left[\frac{-1}{4(4x-2)}\right]_1^k = \frac{1}{10}$

$V = \left[\frac{-1}{16k-8} - \left(\frac{-1}{8}\right)\right] = \frac{1}{10}$

$(-8 + 16k - 8)10 = (16k-8)(8)$

$-80 + 160k - 80 = 128k - 64$

$32k = 96$

$k = 3$

11.

probability student can swim = $\frac{1}{6}$

Probability for at least 2 students can swim

$1 - {}^5C_0 - {}^5C_1$

$1 - {}^5C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 - {}^5C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4$

$1 - (0.401878 - (5)(0.166667)(0.482253))$

$1 - 0.401878 - 0.401878$

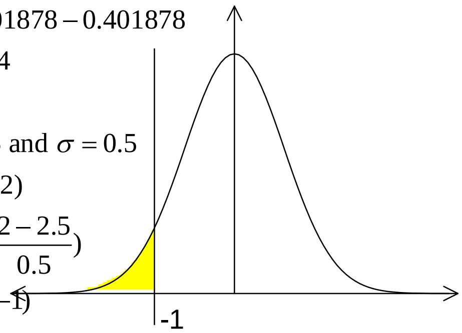
0.19624

$\mu = 2.5$ and $\sigma = 0.5$

$P(X < 2)$

$P(Z < \frac{2 - 2.5}{0.5})$

$P(Z < -1)$

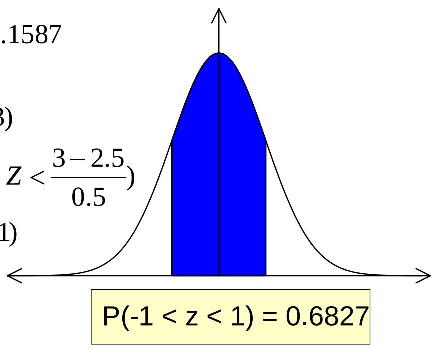


$P(z > 1) = 0.1587$

$P(2 < X < 3)$

$P(\frac{2 - 2.5}{0.5} < Z < \frac{3 - 2.5}{0.5})$

$P(-1 < Z < 1)$



P(durians between 2 and 3) = $\frac{n\text{(durians between 2 and 3)}}{\text{total durians in the orchard}}$

$0.6827 = \frac{3413}{Total}$

Total durians = $\frac{3413}{0.6827} = 4999$ durians

Section C

12.

$v = 3 + 5t - 2t^2$

When P begins to reverse its direction,

$v = 0$

$2t^2 - 5t - 3 = 0$

$(2t + 1)(t - 3) = 0$

$t = 3s$

$s = 3t + \frac{5t^2}{2} - \frac{2t^3}{3} + c$

When $t = 0, s = 0,$ thus $c = 0$

$s = 3t + \frac{5t^2}{2} - \frac{2t^3}{3}$

$s_3 = 3t + \frac{5t^2}{2} - \frac{2t^3}{3}$

When $t = 3s$

$s_3 = 9 + \frac{45}{2} - 18$

$s_3 = 13.5m$

$s_4 = 12 + 40 - 42.6667$

$s_4 = 9.3333...m$ (note* from O)

Total distance travelled = 13.5+4.1667 *m*

17.6667 *m*

When velocity of P reached maximum, a = 0m/s^2

$\frac{dv}{dt} = 5 - 4t = 0$

that happens when $t = \frac{5}{4}s$

Given that acceleration of Q, $a = 4t - 2$

$dv = 4t - 2dt$

$v = \frac{4t^2}{2} - 2t + c$

When $t = 0, v = -3$ and $c = ?$

$v = 2(\frac{25}{16}) - 2(\frac{5}{4}) - 3$

$v = 3.125 - 2.5 - 3$

$v = -2.375m/s$

13.

A	$\frac{6}{4} \times 100 = 150$	$\frac{12}{4} \times 100 = 300$	***a***
B	$\frac{5}{4} \times 100 = 125$	$\frac{8}{4} \times 100 = 200$	1
C	$\frac{12}{10} \times 100 = 120$	$\frac{15}{10} \times 100 = 150$	4
D	$\frac{8}{5} \times 100 = 160$	$\frac{b}{5} \times 100 = 20b$	2
Item	***2001***	***2002***	***w***

$137.5 = \frac{150a + 125 + 480 + 320}{7 + a}$

$962.5 + 137.5a = 150a + 925$

$37.5 = 12.5a$

$a = 3$

$210 = \frac{300a + 200 + 600 + 40b}{7 + a}$

$210 = \frac{900 + 200 + 600 + 40b}{10}$

$2100 - 1700 = 40b$

$b = \frac{400}{40} = 10$

$I_{2004/02}$	$I_{2004/00}$	w	Iw
130	$\frac{15.6}{4}\times 100 = 390$	3	1170
130	$\frac{10.4}{4}\times 100 = 260$	1	260
130	$\frac{19.5}{10}\times 100 = 195$	4	780
130	$\frac{13}{5}\times 100 = 260$	2	520
		$\sum w = 10$	$\sum Iw = 2730$

$$\bar{I}_{04/02} = \frac{(130\times 3 + 130\times 1 + 130\times 4 + 130\times 2)}{10} = \frac{1300}{10}$$

$$\bar{I}_{04} = 130$$

$$\bar{I}_{04/00} = \frac{2730}{10}$$

$$\bar{I}_{04/00} = 273.0$$

14.

$50x + 40y \leq 600$

$5x + 4y \leq 60$.............(1)

$25x + 30y \geq 300$

$5x + 6y \geq 60$..............(2)

$\frac{x}{y} \leq \frac{3}{1}$

$x \leq 3y$

$3y \geq x$.........................(3)

From the graph, the maximum profit is when $x = 4$ and $y = 10$

*(since we are dealing with cars, the values has to be whole numbers)

$8(4) + 10(10) = RM132$

15.

a)

Using sine rule,

$$\frac{15}{\sin 65} = \frac{ED}{\sin 35}$$

$$ED = \frac{15\sin 35}{\sin 65}$$

$$ED = \frac{8.603646}{0.906307} = 9.49308cm$$

b)

Using cosine rule,

$$BD^2 = 9^2 + 14^2 - 2(9)(14)\cos A$$

$$\cos A = \frac{81 + 196 - 360.47427}{252}$$

$$\cos A = -\frac{83.47427}{252} = -0.331242$$

$$A = 109.34^\circ$$

Area of whole diagram = Area of $\Delta ABD + \Delta ECD$

Area of $\Delta ABD = \frac{1}{2}(9)(14)\sin(109.34^\circ)$

Area of $\Delta ABD = 59.4449cm^2$

Area of $\Delta ECD = \frac{1}{2}(9.49308)(15)\sin(80)$

Area of $\Delta ECD = 70.11644cm^2$

$\therefore$ Area of whole diagram $= 59.4449 + 70.11644$

$129.5613cm^2$

End of Marking Scheme
David Ch'ng Y.S.
2008