Solution for Q1

$y = 3 + x$ .... (eq.1)

From $\frac{2}{x} - \frac{x}{y} = 1$ we can rearranged it to

$2y - x^2 = xy$ ....(eq.2)

Sub. eq.1 into eq.2

$2(3+x) - x^2 = x(3+x)$ ← (1)

$6 + 2x - x^2 = 3x + x^2$

$2x^2 + x - 6 = 0$ ← (1)

$(2x-3)(x+2) = 0$ ← (1)

$\therefore x = \frac{3}{2}$ or $x = -2$ ← (1)

For $x = \frac{3}{2}$, sub. it back to eq.1

$y = 3 + \left(\frac{3}{2}\right) = \frac{9}{2}$ ← (1)

For $x = -2$

$y = 3 - 2 = 1$

$\left(\frac{3}{2}, \frac{9}{2}\right)$ or $(-2,1)$ ← (2)

Solution for Q2.

Since O is the origin, its coordinates are (0,0)

Equation of OA

From the diagram, mentally the student can obtained the gradient, $m_{OA} = 2$ ← (1)

Thus the equation of OA is $y = 2x$ ← (1)

Note that the intercept passes through the origin.

BC is pependicular to OA, this means that

$m_{BC} \times m_{OA} = -1$ ← (1)

$m_{BC} = \frac{-1}{2}$ ← (1)

By using taylor's formula, ie. $y - y' = m(x - x')$

Equation of BC at (2,4)

$y - 4 = -\frac{1}{2}(x-2)$ ← (1)

$2y - 8 = -x + 2$

$2y = -x + 10$ or $y = -\frac{1}{2}x + 5$ ← (1)

Coordinates of C * Note that C is on the $x-axis$

When $y = 0, x = 10$

$Hence, C(10,0)$ ← (1)

Solution to Q3.

If $3\frac{1}{2}$ and -2 are roots of $2x^2 + mx - 2n - 4 = 0$

(1) → $(2x-7)(x+2) = 0$

$2x^2 + 4x - 7x - 14 = 0$

(1) → $2x^2 - 3x - 14 = 0$

compares it with $2x^2 + mx - 2n - 4 = 0$

(1) → $-3 = m$ and $-2n - 4 = -14$ ← (1)

$m = -3$ while $2n = 14 - 4$

(1) → $m = -3$ and $n = 5$ ← (1)

Solution for Q4

Given that $y = \frac{6}{x^2}$ or $y = 6x^{-2}$

(1) → $\frac{dy}{dx} = -12x^{-3}$ when $x = 3$

$\frac{dy}{dx} = -12(3)^{-3} = \frac{-12}{27}$

$\frac{dy}{dx} = -\frac{4}{9}$ ← (1)

Find the approximation of $\frac{6}{3.5^2}$

let $x = 4$ and $\delta x = -0.5$ ← (1)

When δx is small, $\frac{dy}{dx} \approx \frac{\delta y}{\delta x}$

Thus, $\delta y \approx -\frac{12}{x^3} \bullet \delta x$ ← (1)

Sub. $x = 4$ and $\delta x = -0.5$

$\delta y = -\frac{12}{64} \times -0.5$ ← (1)

$\delta y = 0.09375$ ← (1)

Therefore, the approximation of $\frac{6}{3.5^2}$ is

$y + \delta y = \frac{6}{x^2} + 0.09375$, when $x = 4$ ← (1)

$y + \delta y = 0.375 + 0.09375 = 0.46875$ ← (1)

Add. Maths p2
marking scheme
mid year 2009

Solution for Q5.

x	$x-\bar{x}$	$(x-\bar{x})^2$	x^2
5	-8	64	25
9	-4	16	81
12	-1	1	144
17	4	16	289
22	9	81	484
		$\sum(x-\bar{x})^2 = 178$ (1)	$\sum x^2 = 1023$ (1)

$$\bar{x} = \frac{5+9+12+17+22}{5} \quad (1)$$

$$\bar{x} = 13 \quad (1)$$

$$Variance, \sigma^2 = \frac{\sum(x-\bar{x})^2}{N} \text{ or } \frac{\sum x^2}{N} - (\bar{x})^2 \quad (1)$$

$$\sigma^2 = \frac{178}{5} \quad (1)$$

$$\sigma^2 = 35.6 \ \therefore \text{ standard deviation, } \sigma = 5.9666 \quad (1)$$

or

$$\sigma^2 = \frac{1023}{5} - 169$$

$$\sigma^2 = 204.6 - 169$$

$$\sigma^2 = 35.6 \ \therefore \text{standard deviation, } \sigma = 5.9666$$

Solution for Q6.

$$\log_5(x+1) - \log_5(3x-1) = -1$$

$$\log_5(x+1) - \log_5(3x-1) = -\log_5 5 \quad (1)$$

$$\log_5 \frac{(x+1)}{3x-1} = \log_5 5^{-1} \quad (1)$$

antilog for $\log_5$ both sides

$$\frac{(x+1)}{3x-1} = \frac{1}{5} \quad (1)$$

$$5x+5 = 3x-1 \quad (1)$$

$$2x = -6$$

$$x = -3 \quad (1)$$

Solution for Q7

$$y = ab^{\frac{x}{2}}$$

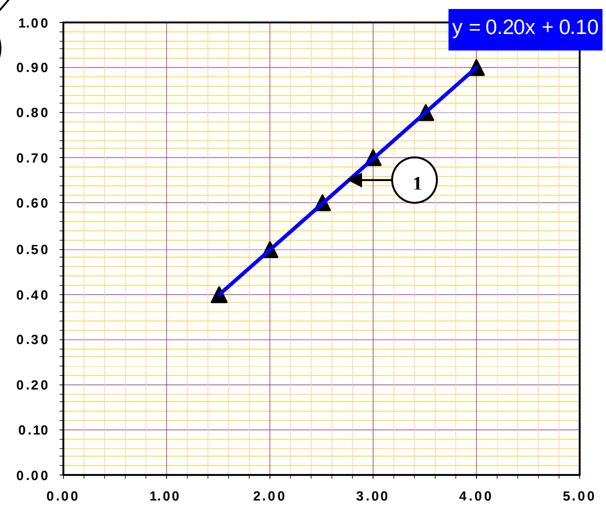
$$\lg y = \lg\left(ab^{\frac{x}{2}}\right) \quad (1)$$

$$\lg y = \lg a + \frac{x}{2}\lg b$$

$$\lg y = \frac{1}{2}\lg b(x) + \lg a \quad (1)$$

x	1.5	2.0	2.5	3.0	3.5	4.0
$\lg y$	0.398	0.499	0.602	0.699	0.799	0.899

(1)



From the graph the gradient is $0.2 = \frac{1}{2}\lg b$ (1)

Therefore, $\lg b = 0.4$

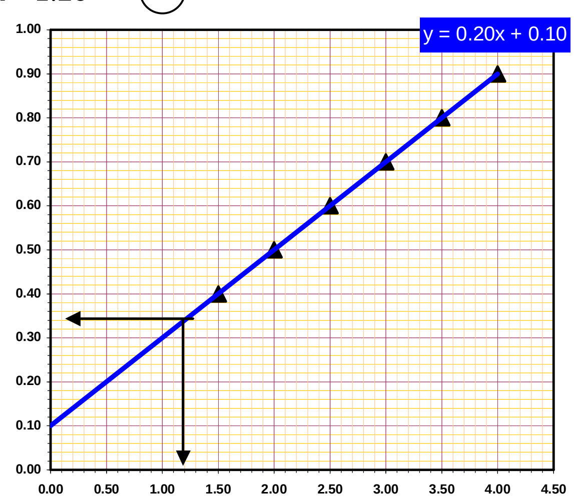
$b = \lg^{-1}(0.4)$

$b = 2.51$ (1)

The intercept $0.1 = \lg a$ (1)

$a = 10^{0.1}$

$a = 1.26$ (1)



When x = 1.2 the intercept is 0.34

$\log a = 0.34$

Thus, $a = 10^{0.34}$

$a = 2.18776$ (1)

Therefore y = 2.19 (1)

Solution for Q8.

Given that $P(-5,0)$ and $Q(3,4)$

$$A(2,k)=\left(\frac{3(m)+(-5)n}{m+n},\frac{3(4)+n(0)}{m+n}\right) \leftarrow (1)$$

By comparison,

$$\frac{3m-5n}{m+n}=2 \leftarrow (1)$$

$$3m-5n=2m+2n$$

$m=7n$ .... let it be eq.1 ← (1)

Also,

$$\frac{12}{m+n}=k \leftarrow (1)$$

$$12=k(m+n)$$

$$12=k(8n)$$

$$k=\frac{3}{2n} \leftarrow (1)$$

$$m_{PQ}=\frac{4-0}{3+5}=\frac{1}{2}$$

$$m_{QA}=m_{PQ}=\frac{1}{2}$$

$$m_{QA}=\frac{4-k}{3-2}=\frac{1}{2} \leftarrow (1)$$

$$8-2k=1$$

$$2k=7$$

$$k=\frac{7}{2} \leftarrow (1)$$

$$\frac{7}{2}=\frac{3}{2n}$$

$$n=\frac{3}{7} \leftarrow (1)$$

$$m=7\left(\frac{3}{7}\right)$$

$$m=3 \leftarrow (1)$$

$$m:n=3:\frac{3}{7}$$

$$m:n=7:1 \leftarrow (1)$$

Solution for Q9

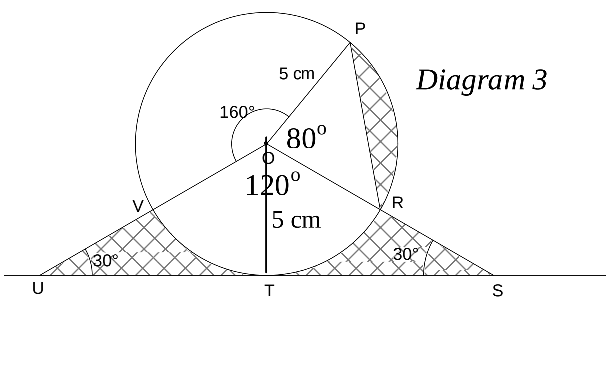


Diagram 3

Area of shaded region = area of segment OPR + (area of triangle OUS- area of sector OVTR)

$$Tan30^{o}=\frac{OT}{TS} \text{ or } \frac{OT}{UT} \leftarrow (1)$$

$$\frac{1}{\sqrt{3}}=\frac{5}{TS} \text{ or } TS=5\sqrt{3}=8.66603 \text{ unit}=US \quad (1)$$

$$\therefore \text{ Area of } \Delta OUS=\frac{1}{2}(2\bullet 8.66603)(5) \leftarrow (1)$$

$$\text{Area of } \Delta OUS=43.33015 \text{ unit}^2 \leftarrow (1)$$

$$\text{Area of Sector } OVTR=\frac{1}{2}(5^2)2.09467 \leftarrow (1)$$

$$\theta=2.09467rad$$

$$\text{Area of Sector } OVTR=26.18335 \text{ unit}^2 \leftarrow (1)$$

$$\text{Area of Segment }=\frac{1}{2}(5^2)(1.3964-0.8660) \quad (1)$$

$$\text{Area of Segment}=6.63 \text{ unit}^2 \leftarrow (1)$$

$\therefore$ Area of the shaded regions = A

$$\text{A}=6.63+43.33015-26.18335 \leftarrow (1)$$

$$\text{A}=23.7767 \text{ unit}^2 \leftarrow (1)$$

Solution for Q10.

$$\int_0^1 (x-1)(\sqrt{x}-3)dx$$

$$\int_0^1 \left(x\sqrt{x}-3x-\sqrt{x}+3\right)dx \leftarrow (1)$$

$$\int_0^1 \left(x^{\frac{3}{2}}-3x-x^{\frac{1}{2}}+3\right)dx \leftarrow (1)$$

a) $$\left[\frac{2x^{\frac{5}{2}}}{5}-\frac{3x^2}{2}-\frac{3x^{\frac{3}{2}}}{2}+3x\right]_0^1 \leftarrow (1)$$

$$\left(\frac{2}{5}-\frac{3}{2}-\frac{3}{2}+3\right)-(0) \leftarrow (1)$$

$$\frac{2}{5}-\frac{6}{2}+\frac{6}{2}$$

$$=\frac{2}{5} \leftarrow (1)$$

b) Continuation from Q10….

$Area, A=\int_0^7 (x+9)^{\frac{1}{2}}dx$

$A=\left[\frac{2(x+9)^{\frac{3}{2}}}{3}\right]_0^7$ ← (1)

$A=\left(\frac{2(16)^{\frac{3}{2}}}{3}\right)$ ← (1)

$A=\frac{2(4^2)^{\frac{3}{2}}}{3}$ ← (1)

$A=\frac{2(64)}{3}$

$A=42.667 \text{ unit}^2$ ← (1)

Total Area of the shaded region 2A
85.334 unit^2. ← (1)

Solution for Q11.

$h=96t-16t^2$

$\frac{dh}{dt}=96-32t$ ← (1)

(1) → When the ball reaches its maximum height, $v=0$

$96-32t=0$ ← (1)

$32t=96$

$t=3s$ ← (1)

Sub. the value of $t=3s$ to $h=96t-16t^2$

$h=96(3)-16(9)$ ← (1)

$h=288-144$

$h=144m$ ← (1)

$h=96t-16t^2$

When the ball falls to the ground, $h=0$

$t(96-16t)=0$ ← (1)

$t=0$ or $t=6s$ ← (1)

Its decceleration just before the ball hits the ground

$\frac{d^2h}{dt^2}=a=-32$ ← (1)

From the formula the accleration is independent of time, t

$Hence, a=32m/s^2$ ← (1)

Solution for Q13

If $\sin PSR=\frac{3}{4}$ and it is an obtuse

$\measuredangle\text{PSR}=\sin^{-1}(0.75)$ ← (1)

$\measuredangle\text{PSR}=180-48.59$ ← (1)

$\measuredangle\text{PSR}=131.41^{\circ}$ ← (1)

Using cosine rule,

$PR^2=81+49-2(9)(7)\cos 131.41$ ← (1)

$PR^2=130-126(-0.66144)$ ← (1)

$PR^2=130+83.34144=213.34144$ ← (1)

(1) → $PR=14.61cm$ correct to 2 decimal places.

Using sine rule:

$\frac{\sin 10}{4}=\frac{\sin\angle PQR}{PR}$ ← (1)

$\frac{0.17365}{4}=\frac{\sin\angle PQR}{14.61}$ ← (1)

$\sin\angle PQR=0.63426$

$\angle PQR=39.37^{\circ}$ ← (1)

Solution for Q14.

Given $\lg 2=0.3010, \lg 3=0.4771$ and $\lg 5=0.6990$

$a)\ \lg 12.5=\lg(\frac{125}{10})$ ← (1)

$\lg\left(\frac{25}{2}\right)=2\lg 5-\lg 2$ ← (1)

$2(0.6990)-0.3010$

1.097 ← (1)

$b)\ \lg 1.2=\lg\left(\frac{12}{10}\right)$ ← (1)

$\lg 6-\lg 5$ ← (1)

$\lg 3+\lg 2-\lg 5$ ← (1)

$0.4771+0.3010-0.6990$

0.0791 ← (1)

$c)\ \lg\left(\frac{10}{9}\right)=\lg 10-\lg 9$ ← (1)

$1-2\lg 3$

$1-2(0.4771)$ ← (1)

$1-0.9542$

0.0458 ← (1)

Solution for Q15.

A straight line, L is $\perp$ to line $2y=-4x+1$

$or\ y=-2x+\frac{1}{2}$ ← (1)

$Thus,$ gradient of $L=\frac{1}{2}$ ← (1)

Equation of L passing through $(3,-2)$

$y+2=\frac{1}{2}(x-3)$ ← (1)

$2y+4=x-3$ ← (1)

$2y=x-7$ or $y=\frac{1}{2}x-\frac{7}{2}$ ← (2)

Intercept of x, when $y=0$

$x=7$ ← (1)

$(7,0)$ ← (1)

Intercept of y, when $x=0$

$y=-\frac{7}{2}$ ← (1)

$(0,-\frac{7}{2})$ ← (1)

The inequalities are:

$x+y\le 60$

$3y\ge 2x$ ← (3)

$y-x\le 30$

a) If the number of piano students equals the number of guitar students,

$y=x$

Draw a line $y=x$ and see the maximum point touched in the region R.

From the graph, $x=30, y=30$ ← (1)

b) Draw a profit line $120x+80y=k$

Example: $120x+80y=960$

Slide up until the point where its intercept is the highest. (1)

From the graph, that point is $x=36, y=24$

Sub. it in $120(36)+80(24)=6240$

Thus, maximum profit is RM 6240 ← (1)

Solution for Q12.