Add Mathematics Paper 2  
Mid Year Examination  
2012

Section A  
[40 marks]

Answer all questions in this section

1. Solve the following simultaneous equations:
   \[4x + 4y = x^2 - xy + y^2 - 9 = 12\]  
   [5 marks]

2. The minimum value of the function \(f(x) = 2x^2 - 3x + m\) is \(\frac{7}{8}\)
   a) Find the value of \(m\) by completing the squares.  
      [3 marks]
   b) Sketch the graph of the function \(y = f(x)\)  
      [3 marks]

3. A function \(g\) is defined as \(g(x) = x^2 + mx + n\). Given \(g(4) = 30\) and \(g(-1) = 0\)
   a) Determine the values of \(m\) and of \(n\)  
      [3 marks]
   b) Hence, determine the value of \(g(2)\)  
      [2 marks]

4. 100 students take a test which consists of 40 questions with 1 mark for each correct response. 
   Table 1 shows the distribution on the marks obtained.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>12</td>
<td>20</td>
<td>27</td>
<td>19</td>
<td>12</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

   a) Calculate the
   i) mean marks  
      [5 marks]
   ii) inter-quartile range of the distribution
   b) If 70% of the students pass the test, calculate the passing marks.  
      [3 marks]

5. Given that \(\overrightarrow{OA} = -2pi + (1 - p)j, \overrightarrow{OB} = -4pi + (5 - p)j, \overrightarrow{OC} = (10 - p)i\) and \(M\) is the mid-point of \(AB\)
   a) Express \(\overrightarrow{CM}\) in terms of \(p\)  
      [3 marks]
   b) Calculate the values of \(p\) if \(|\overrightarrow{OM}| = |\overrightarrow{CM}|\)  
      [4 marks]

6. \(\text{Diagram 1}\) shows a square \(PQRS\) which is divided into four smaller congruent squares. One of these smaller squares is shaded and labelled as \(A_1\). The same process is performed on one of the smaller squares and the corresponding smaller square is labelled as \(A_2\) as shown. The process is repeated to obtain \(A_3, A_4, A_5, \ldots A_n\). If \(PQ\) is 10 units, calculate
   a) \(A_5\)  
      [2 marks]
   b) the sum of the areas of the first 5 shaded  
      [2 marks]
   c) sum of the areas of the shaded squares when the process is repeated infinitely.  
      [2 marks]
Section B  
[40 marks]
Answer four questions from this section

7. Use graph paper to answer this question.  
Table 2 shows the values of two variables $x$ and $y$ obtained from an experiment. It is know that $x$ and $y$ are connected by the formula $y = ab^{-x}$ where $a$ and $b$ are constants.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>39.8</td>
<td>18.8</td>
<td>4.78</td>
<td>1.22</td>
<td>0.60</td>
<td>0.30</td>
</tr>
</tbody>
</table>

a) Explain how a straight line can be drawn to represent the above data [2 marks]  
b) Using a scale of 2 cm to 1 unit on the horizontal axis and 2 cm to 0.2 unit on the vertical axis, draw the graph of the above best straight line. [4 marks]  
c) From the graph, estimate the values of $a$ and of $b$. [4 marks]

8. Diagram 2 shows a parallelogram $ABCD$. $A$ and $B$ are points $(1, 5)$ and $(4, 2)$ respectively. The diagonals $AC$ and $BD$ intersect at $M$. Given $BD$ is parallel to the line $5x + y = 0$ and $AC$ is perpendicular to the line $5x - y = 0$.

a) Find the equations of $BD$ and $AC$ [3 marks]  
b) Find the coordinates of $M$ [3 marks]  
c) Find the coordinates of $C$ [2 marks]  
d) Calculate the area of parallelogram $ABCD$. [2 marks]

9. Diagram 3 shows two circles with radius 3 cm and 1 cm touching each other at $Q$. Given $O$ and $P$ are the centres of the circles; $ARB$ and $QR$ are common tangents to the two circles.

a) Show that $\angle POA = \frac{1}{3} \pi$ rad [2 marks]  
b) Calculate  
   i) the area of the shaded region [4 marks]  
   ii) perimeter of the shaded region [4 marks]

10. Diagram 4 shows a curve $y = x(x - 3)^2$ intersect the straight line $y = 4x$ at the points $O$, $P$ and $Q$.

   a) Find the coordinates of $P$ and $Q$ [7 marks]  
   b) When the region bounded by the curve $y = \frac{x}{2}$, the y-axis, and the lines $y = 2$ and $y = k$ is rotated through $360^\circ$ about the y-axis, the volume of the solid generated is $\frac{3}{4} \pi$ unit$^3$. Find the value of $k$ where $k > 2$. [3 marks]
11. a) John has registered 10 subjects in the school examination. The probability that John will obtain grade A in a subject is 0.8. The school has decided to award Certificates of Excellence to students who have obtained grade A in at least 9 subjects in the examination.
i) Find the probability that John will be awarded the Certificate of Excellence for the first term examination.
ii) For the 3 terminal examinations, a student who has obtained the Certificate of Excellence more than once will be awarded the Medal of Excellence. Find the probability that John will be awarded the Medal of Excellence. [5 marks]
b) The life spans of car batteries produced by a factor are found to be normally distributed with a mean of 28 months and a standard deviation of 6 months.
i) Find the probability that a battery chosen at random from the factory has a life span of more than 3 years.
ii) A defective battery within one year will be replaced free of charge. If the factory produces 10,000 units of batteries, estimate the number of batteries need to be replaced. [5 marks]

Section C
[20 marks]
Answer two questions from this section

<table>
<thead>
<tr>
<th>Item</th>
<th>Index Number</th>
<th>Weightage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetables</td>
<td>115</td>
<td>4</td>
</tr>
<tr>
<td>Egg</td>
<td>130</td>
<td>1</td>
</tr>
<tr>
<td>Fish</td>
<td>110</td>
<td>5</td>
</tr>
<tr>
<td>Meat</td>
<td>150</td>
<td>x</td>
</tr>
</tbody>
</table>

12. Table 3 shows the price indices and their corresponding weightages for four types of food in 2004, taking 2000 as the base year.
a) Calculate the price of an egg in the year 2000 if the price of an egg in year 2004 is 26 sen. [2 marks]
b) Given that the composite index representing the year 2004 based on the year 2000 is 120, calculate
i) the value of x, [4 marks]
ii) the monthly expenses on the above items in year 2000 if the monthly expenses on the items in 2004 was RM 1500 [2 marks]
c) The expenses on the above food items increase by 20% from the year 2004 to 2008. Find the composite index in year 2008 based on the year 2000 [2 marks]
13. a) In a triangle $ABC$, $AB = 10 \text{ cm}$, $BC = 8 \text{ cm}$ and $\angle BAC = 38^\circ$.
   i) Find the possible values of $\angle BCA$ [3 marks]
   ii) When the length of $AB$ and the size of $\angle BAC$ are fixed at 10 cm and $38^\circ$, find the least value of length $BC$. [2 marks]

b) [Diagram 5]

Diagram 5 shows a pyramid with rectangular base $PQRS$. $T$ is vertically above $S$ such that $TS = 3 \text{ cm}$. Given that $PQ = 2 \text{ cm}$, $QR = 4 \text{ cm}$, $U$ and $V$ are mid-points of $PQ$ and $QR$ respectively. Calculate the area of triangle $TUV$ [5 marks]

14. A particle $X$ moves along a straight line and passes through a fixed point $O$, with velocity, $6 \text{ ms}^{-1}$, its acceleration, $a$, $t$ seconds after passing through a fixed point $O$, is given by $a = 4 - 4t$. The particle changes its direction of motion at point $A$.
   a) Find
      i) the time when particle $X$ is at $A$, [7 marks]
      ii) the total distance travelled by particle $X$ during the first 5 seconds

b) A particle $Y$ moves in the same straight line with velocity $v_y \text{ ms}^{-1}$. Its velocity is given by $v_y = t^2 - 7t + 12$ where $t$ seconds after passing through $O$. Determine whether particles $X$ and $Y$ move in the same or opposite direction when $t = 2s$. [3 marks]

15. Diagram 6 shows that $\overrightarrow{PQ} = 4i - 6j$, $\overrightarrow{PR} = 2i + 4j$, and $S$ is a point on the line $QR$ such that $QS = 3SR$.
   a) Find each of the following vectors in terms of $i$ and $j$
      i) $\overrightarrow{QR}$ [4 marks]
      ii) $\overrightarrow{QS}$
      iii) $\overrightarrow{PS}$

b) $PS$ is extended to $T$ so that $PT = 2PS$, find $|\overrightarrow{PT}|$ [4 marks]

c) If $U$ is a point on $QR$ and $\overrightarrow{SU} = m\overrightarrow{i} - 4\overrightarrow{j}$, find the value of $m$. [2 marks]

END OF QUESTIONS